

NEUTRINO MASS-AN OVERVIEW

R. N. Mohapatra^{a*}

^aDepartment of Physics, University of Maryland, College Park, MD-20742, USA

A brief overview of the present status of neutrino mass physics is given.

1. INTRODUCTION

There is now convincing evidence from solar, atmospheric, accelerator as well as reactor neutrino studies that neutrinos, long thought to be massless, are indeed massive and like the quarks, they mix among themselves leading to the phenomenon of neutrino oscillations. Since the standard model predicts that neutrinos are massless, this is the first conclusive evidence for physics beyond the standard model and as such, has led to a new and important phase in the exploration of physics beyond the TeV scale. In this talk, I will present a brief overview of our present understanding of neutrino properties, what the recent discoveries have taught about new physics and where we go from here.

Throughout this report, we will use the notation, where the flavor or weak eigenstates are denoted by ν_α (with $\alpha = e, \mu, \tau, \dots$) and they are expressed in terms of the mass eigenstates ν_i ($i = 1, 2, 3, \dots$) as follows: $\nu_\alpha = \sum_i U_{\alpha i} \nu_i$. The $U_{\alpha i}$, the elements of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix represent the observable mixing angles in the basis where the charged lepton masses are diagonal. In any other basis, one has $U = U_\ell^\dagger U_\nu$, where the matrices U_ℓ and U_ν are the matrices that diagonalize the charged lepton and neutrino mass matrices respectively. In the rest of this talk, we will assume that neutrinos are Majorana particles; even though this has not been experimentally established (and indeed, one of the major goals of experimental efforts in neutrino physics is to confirm or refute this), theoretical discussions are

more convenient in this case and also most theoretical models predict neutrinos to be Majorana particles.

For the case of three Majorana neutrinos, the PMNS matrix U can be written as: VK where V is the following matrix

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \quad (1)$$

and $K = \text{diag}(1, e^{i\phi_1}, e^{i\phi_2})$. There are three angles and three phases that characterize the mixings. In addition there the three masses. It is the goal of the planned and current experiments to determine these 9 parameters as accurately as possible.

In this review, I first summarize our present state of understanding of these various parameters and then proceed to give an overview of what kind of new physics is implied by these data.

2. WHAT WE KNOW NOW:

2.1. Mass difference squares and Mixings

Thanks to Super-Kamiokande results on the atmospheric neutrinos, the results for solar neutrinos from Chlorine, Super-Kamiokande SAGE, GALLEX and most recently from the SNO experiment on both charged and neutral currents, as well as the KAMLAND, K2K and CHOOZ-PALO-Verde results, there now appears to be a rough outline of the pattern of mixings among the various neutrinos[1]. In terms of the parameters defined above, it seems clear that both θ_{12} and θ_{23} are large and the angle θ_{13} is small. This is in sharp contrast with the corresponding mixings in the quark sector, which are all small.

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Oscillations also have given us the approximate values of the mass difference squares for the neutrinos. The present allowed values for the θ_{ij} as well as the Δm^2 's are (at 3σ level): $\sin^2 2\theta_{23} \geq 0.92$; $1.2 \times 10^{-3} \text{eV}^2 \leq \Delta m_{23}^2 \leq 4.8 \times 10^{-3} \text{eV}^2$; $0.70 \leq \sin^2 2\theta_{12} \leq 0.95$; $5.4 \times 10^{-5} \text{eV}^2 \leq \Delta m_{12}^2 \leq 9.5 \times 10^{-5} \text{eV}^2$; $\sin^2 \theta_{13} \leq 0.23$. There is no information on any of the phases right now.

Since the oscillation data gives only the mass difference squares, it allows for three possible arrangements of the different mass levels:

- (i) Normal hierarchy i.e. $m_1 \ll m_2 \ll m_3$. In this case, we can deduce the value of $m_3 \simeq \sqrt{\Delta m_{23}^2} \simeq 0.03 - 0.07 \text{ eV}$. In this case $\Delta m_{23}^2 \equiv m_3^2 - m_2^2 > 0$. The solar neutrino oscillation involves the two lighter levels. The mass of the lightest neutrino is unconstrained. If $m_1 \ll m_2$, then we get the value of $m_2 \simeq 0.008 \text{ eV}$.
- (ii) Inverted hierarchy i.e. $m_1 \simeq m_2 \gg m_3$ with $m_{1,2} \simeq \sqrt{\Delta m_{23}^2} \simeq 0.03 - 0.07 \text{ eV}$. In this case, solar neutrino oscillation takes place between the heavier levels and we have $\Delta m_{23}^2 \equiv m_3^2 - m_2^2 < 0$.
- (iii) Degenerate neutrinos i.e. $m_1 \simeq m_2 \simeq m_3$.

It is hoped that future long baseline experiments as well as the searches for neutrinoless double beta decay can resolve between the different possibilities. We will discuss this below.

2.2. Overall scale for masses

Oscillation experiments do not tell us about the overall scale of masses. There are several ways to pin down this scale:

2.2.1. Neutrino mass from beta decay

(i) One way is to directly search for the effect of nonzero neutrino mass in the beta decay spectrum by looking for structure near the end point of the electron energy spectrum. (A commonly used nucleus is tritium.) It will measure a mass regardless of whether neutrino is a Dirac or Majorana particle. In this case, one measures the quantity $m_\beta \equiv \sqrt{\sum_i |U_{ei}|^2 m_i^2}$. The Troitsk

and Mainz results put the present upper limit on $m_\beta \leq 2.2 \text{ eV}$ [2]. The proposed KATRIN experiment is projected to lower the sensitivity down to 0.2 eV , which will have important implications for the theory of neutrino masses. For instance, if the result is positive, it will imply a degenerate spectrum; on the other hand a negative result will be a very useful constraint.

2.2.2. Neutrino mass from neutrinoless double beta decay

Second way is to search for neutrinoless double beta decay, $\beta\beta_{0\nu}$, which can proceed if there is a sizeable Majorana mass for the neutrino or if there are lepton number violating interactions[3]. In the first case, one measures the quantity $m_{ee} = \sum U_{ei}^2 m_i$. In the second case, the neutrino necessarily has a Majorana mass[4]; however, if the resulting induced mass is small, then neutrinoless double beta decay will measure the strength of the new interactions (such as doubly charged Higgs fields or R-parity violating interactions etc) rather than neutrino mass. There are many examples of models where new interactions can lead to $\beta\beta_{0\nu}$ decay rate in the observable range without at the same time giving a significant Majorana mass for the neutrinos. As a result, one must be careful in interpreting any nonzero signal in a $\beta\beta_{0\nu}$ experiment and not jump to a conclusion that a direct measurement of neutrino mass has been made. Way to tell whether such a nonzero signal is neutrino mass or is a reflection of new interactions is to supplement $\beta\beta_{0\nu}$ decay results with collider searches for the new interactions. Thus collider experiments such as those at LHC and the double beta experiments play complementary role.

Present upper limits on $\beta\beta_{0\nu}$ decay lifetimes from the Heidelberg-Moscow and can be translated to an upper limit on $m_{ee} \leq 0.3 \text{ eV}$ or so. There is a claim of a discovery of neutrinoless double beta decay in the enriched ^{76}Ge experiment by the Heidelberg-Moscow collaboration[5]. Interpreted in terms of a Majorana mass of the neutrino, this implies m_{ee} between 0.11 eV to 0.56 eV . If confirmed, this result is of fundamental significance. There have been much discussion of this in literature[6].

2.2.3. Cosmology and neutrino mass

A very different way to get information on the absolute scale of neutrino mass is from the study of cosmic microwave radiation spectrum as well as the study of large scale structure in the universe. A rough way this comes about is that if neutrinos are present in abundance in the universe at the epoch of structure formation with a significant mass, it will affect structure formation. For instance, for a given neutrino mass m , all structures on a scale smaller than a value given by the inverse of neutrino mass will be washed away by neutrino free streaming. This will reduce power on smaller scales. Thus accurate information of the galaxy power spectrum for small scales can help constrain neutrino mass. Recent results from the WMAP has put a limit on the sum of neutrino masses $\sum m_i \leq 0.7 - 2$ eV[7,8]. More recent results from SDSS sky survey has put a limit of $\sum m_i \leq 1.6$ eV. Hannestad[8] has emphasized that these upper limits can change if there are more neutrino species- e.g. for 5 neutrinos, $\sum m_i \leq 2.12$ eV if they are in equilibrium at the epoch of BBN.

A point worth emphasizing is that the above result is valid for both a Majorana and a Dirac neutrino.

These limits are already in the interesting range: for instance if the limit 0.7 eV is taken seriously, it would imply that each individual neutrino must have an upper limit on its mass of 0.23 eV, which is same as the projected value from the proposed KATRIN experiment. All these limits are going to be much smaller once PLANCK satellite observations are carried out, thereby providing a completely independent source of information on neutrino masses. Furthermore, these results also have implications for models of sterile neutrinos that attempt to explain the LSND results.

2.3. Sterile neutrinos

Another question of great importance in neutrino physics is the number of neutrino species. Measurement of the invisible Z-width in LEP-SLC experiments tell us that only three types of neutrinos couple to the W and Z boson. They correspond to the three known neutrinos $\nu_{e,\mu,\tau}$. This

implies that if there are other neutrino species, then they must have little or no interaction with the W and Z. They are called sterile neutrinos. So the question is: are there any sterile neutrinos and if there are how many there are ?

2.3.1. LSND and sterile neutrinos

The first need for sterile neutrinos came from attempts to explain[12] Los Alamos Liquid Scintillation Detector (LSND) experiment[9], where neutrino oscillations both from a stopped muon (DAR) as well as the one accompanying the muon in pion decay (known as the DIF) have apparently been observed. The evidence from the DAR is statistically more significant and is an oscillation from $\bar{\nu}_\mu$ to $\bar{\nu}_e$. The mass and mixing parameter range that fits data is:

$$\Delta m^2 \simeq 0.2 - 2 \text{eV}^2; \sin^2 2\theta \simeq 0.003 - 0.03 \quad (2)$$

There are points at higher masses specifically at 6 eV² which are also allowed by the present LSND data for small mixings. KARMEN experiment at the Rutherford laboratory has very strongly constrained the allowed parameter range of the LSND data[10]. Currently the Miniboone experiment at Fermilab is under way to probe the LSND parameter region[11].

Since this Δm_{LSND}^2 is so different from that $\Delta m_{\odot,A}^2$, the simplest way to explain these results is to add one[12] or two[13] sterile neutrinos. For the case of one extra sterile neutrino, there are two scenarios: (i) 2+2 and (ii) 3+1. In the first case, solar neutrino oscillation is supposed to be from ν_e to ν_s . This is ruled out by SNO neutral current data. In the second case, one needs a two step process where ν_μ undergoes indirect oscillation to ν_e due to a combined effect of $\nu_\mu - \nu_s$ and $\nu_e - \nu_s$ mixings (denoted by $U_{\mu,s}$ and U_{es} respectively, rather than direct $\nu_\mu - \nu_e$ mixing. As a result, the effective mixing angle in LSND is given by $4U_{es}^2 U_{\mu s}^2$ and the measured mass difference is given by that between $\nu_{\mu,e} - \nu_s$ rather than $\nu_\mu - \nu_e$. This scenario is constrained by the fact that sterile neutrino mixings are constrained by two sets of observations: one from the accelerator searches for nu_μ and ν_e disappearance[14] and the second from big bang nucleosynthesis[15].

The bounds on U_{es} and $U_{\mu s}$ from accelerator

experiments such as Bugey, CCFR and CDHS are of course dependent on particular value of $\Delta m_{\alpha s}^2$ but for a rough order of magnitude, we have $U_{es}^2 \leq 0.04$ for $\Delta m^2 \geq 0.1 \text{ eV}^2$ and $U_{\mu s}^2 \leq 0.2$ for $\Delta m^2 \geq 0.4 \text{ eV}^2$ [16].

It is worth pointing out that SNO neutral current data has ruled out pure $\nu_e - \nu_s$ transition as an explanation of solar neutrino puzzle by 8σ 's; however, it still allows as much as 40% admixture of sterile neutrinos and as we will see below, the sterile neutrinos could very well form a sub-dominant component in solar neutrino transitions.

2.3.2. BBN and sterile neutrinos

Big bang nucleosynthesis (BBN) will put bounds on how many extra neutrinos are allowed by the present observations of primordial abundances of light elements. It is important to remember that the mere existence of a sterile neutrino does not conflict with BBN results. It is effective only if its mass and mixings with active neutrinos satisfy the constraint[15]

$$\Delta m^2 \sin^4 2\theta \geq \xi 10^{-5} \text{ eV}^2. \quad (3)$$

where ξ is a number of order one and is flavor dependent. If for example one interprets the LSND result in terms of a sterile neutrino, one has $\sin^2 2\theta \simeq 10^{-3}$ and $\Delta m^2 \sim \text{eV}^2$ which satisfies the above condition and therefore such a sterile neutrino will count as one extra species of neutrino. We therefore need to discuss whether an extra neutrino species is allowed by light element production at BBN epoch.

The light element abundances at BBN epoch depend on several factors such as the baryon to photon ratio, chemical potential of the neutrinos which measures the excess of neutrinos over anti-neutrinos (or lepton asymmetry of the Universe) and the number of neutrinos in equilibrium with radiation which determines the Hubble expansion rate at that epoch. In generic models, one expects the lepton asymmetry to be of order of the baryon asymmetry of the universe, in which case it has no effect on BBN. Under this assumption, one can derive limits on ΔN_ν , the number of sterile neutrinos from BBN, by using as input the primordial He^4 abundance (denoted as Y_p) and the deu-

terium abundance D/H . The word ‘‘primordial’’ here is crucial. Since the observed abundances may have undergone some modification due to the age of the universe (e.g. stellar processing etc), uncertainties can creep in when one derives the primordial abundances from observed abundances. That this could be so for Y_p has been noted by many authors[15]. In particular, many people have noted that Y_p suffers from large systematic errors. The D/H ratio on the other hand is believed to have less systematic uncertainties although it is somewhat statistics limited. In any case, if both the presently inferred values of Y_p and D/H are taken as inputs, the best fit point turns out to be for $\Delta N_\nu \leq 0$ [17] and the most likely value of $\eta = 5.7 \times 10^{-10}$.

The WMAP experiment has now determined the value of $\eta = (6.14 \pm 0.25) \times 10^{-10}$. One may therefore take this highly precise value of η and try to combine it with the D/H observations to constrain the number of neutrinos leaving aside the Y_p value. This allows for as many as two extra neutrinos[18].

In any case, if the Helium data becomes more precise with less systematic errors and there is independent evidence for sterile neutrinos, then this would be a clear indication for different kind of new physics. One possibility is there is a majoron[19] coupled to neutrinos that contributes significantly to matter effect at the era of BBN to suppress the sterile to active neutrino mixing[20]. This can give rise to a plethora of new phenomena of both laboratory and astrophysical interest[21].

Let us briefly discuss the implications of this discussion for interpretation of the LSND results in terms of sterile neutrinos. We remind the reader that there are three possible sterile neutrino scenarios for LSND: (i) 2+2[12]; (ii) 3+1[22] and (iii) 3+2[13]. It appears that 2+2 models are disfavored by a combination of accelerator as well as solar and atmospheric neutrino data. The 3+1 scenario is however marginally allowed for only specific mass and mixing values. The only one that is consistent with the WMAP data is the one with $\Delta m^2 = 0.8 \text{ eV}^2$ and $\sin^2 2\theta = 2 \times 10^{-3}$. On the other hand the 3+2 scenario requires two sterile neutrinos one of which has a mass around

1 eV and a second one around 4.5 eV. Both have mixings with $\nu_{e,\mu}$ which bring them to equilibrium at the epoch of nucleosynthesis. This scenario would then appear to be in conflict with mass bounds on $\sum m_i$ from WMAP[8].

Recently, another possibility for the existence a sterile neutrino has been suggested in ref.[23]. It was noted in this paper that the now favored LMA solution to the solar neutrino puzzle runs into two possible difficulties: (i) it predicts the Argon production rate which higher than observations at 2σ level (LMA prediction is 3 SNU's as against the observed value of 2.56 ± 0.23) and (ii) SNO data does not show a rise in the low energy region that is predicted by the LMA solution. Both these difficulties can be resolved if there is a sterile neutrino with $\nu_e - \nu_s$ $\Delta m^2 \simeq (0.2 - 2) \times 10^{-5}$ eV² and mixing angle $\sin^2 2\alpha \simeq 10^{-3} - 10^{-5}$. Such a sterile neutrino escapes all the above cosmological bounds and is therefore quite acceptable.

2.4. CP violation

It is clear from Eq. (1) that for Majorana neutrinos, there are three CP phases that characterize neutrino mixings and a complete understanding of leptonic mixing will be incomplete without a knowledge of these phases. There are two possible ways to explore CP phases: (i) one way is to via the long baseline experiments and looking for differences between neutrino and anti-neutrino survival probabilities[24]; (ii) another way is to use possible connection with cosmology. It has often been argued that neutrinoless double beta decay may also provide a alternative way to explore CP violation. A detailed discussion of these issues is beyond the scope of this article. For some discussions, see [25].

3. IMPLICATIONS FOR PHYSICS BEYOND THE STANDARD MODEL:

These discoveries involving neutrinos, which have provided the first evidence for physics beyond the standard model, have raised a number of challenges for theoretical physics. Foremost among them are, (i) an understanding of the smallness of neutrino masses and (ii) under-

standing the vastly different pattern of mixings among neutrinos from the quarks. Specifically, a key question is whether it is possible to reconcile the large neutrino mixings with small quark mixings in grand unified frameworks suggested by supersymmetric gauge coupling unifications that unify quarks and leptons.

3.1. Seesaw mechanism for small neutrino masses: type I and type II seesaw

The first challenge posed by neutrinos, i.e. the extreme smallness of neutrino masses is elegantly answered by the seesaw mechanism [26] which requires an extension of the standard model that includes heavy right handed neutrinos. The light neutrino mass matrix obtained by integrating out heavy right-handed neutrinos is given by

$$M_\nu = -M_\nu^D M_R^{-1} (M_\nu^D)^T, \quad (4)$$

where M_ν^D is the Dirac neutrino mass matrix and M_R is the right-handed Majorana mass matrix.

There are several reasons why the seesaw mechanism is appealing: (i) first, it restores quark-lepton symmetry to the standard model; (ii) secondly, it allows B-L to be an anomaly free gaugeable symmetry, thereby expanding the minimal electroweak gauge group to the the left-right symmetric group $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$, which is known to provide a new way to understand the observed parity violation in weak interactions. In the left-right symmetric model, the electric charge formula is given by[27]:

$$Q = I_{3L} + I_{3R} + \frac{B-L}{2} \quad (5)$$

First of all unlike the corresponding formula in the standard model,, this formula involves only physical quantities like weak isospin and B-L quantum numbers. Secondly, taking variation of both sides of this charge equation above the weak scale, we get $\Delta I_{3R} \simeq -\frac{\Delta(B-L)}{2}$. For purely leptonic processes since $\Delta B = 0$ and weak interactions have parity violation, one must have lepton number violation. In particular, it implies that neutrino in this theory is naturally a Majorana particle. The presence of the heavy right handed neutrino also opens up a new way to understand the origin of matter in the universe from baryogenesis via leptogenesis arising from the decay of

the right handed neutrinos in combination with CP violation.

The above formula for the neutrino mass matrix is called type I seesaw formula. The right-handed Majorana mass scale, M_R , is more or less determined by the mass squared difference needed to understand the atmospheric neutrino data to be around 10^{14} GeV, (if we assume that the Dirac neutrino mass is same as up-type quark mass). Note that this scale is tantalizingly close to the GUT scale suggesting that grand unified theories may provide a natural framework to unravel the mysteries of neutrino physics. In this talk I will strengthen this argument by providing a simple model where this happens. The seesaw formula in Eq. (1) also implies that the scale at which the left-right symmetric gauge group manifests itself is near the GUT scale. One must then look for a GUT group that contains $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. SO(10) happens to be the minimal such group. We would therefore explore to what extent we can understand the properties of the neutrinos within the SO(10) group.

Before proceeding further, let us note a general phenomenon that when the theory containing the N_R becomes parity symmetric, the seesaw formula changes and becomes:

$$M_\nu^{\text{II}} = M_L - M_\nu^D M_R^{-1} (M_\nu^D)^T, \quad (6)$$

where $M_L = f v_L$ and $M_R = f v_R$, where $v_{L,R}$ are the vacuum expectation values of Higgs fields that couple to the right and lefthanded neutrinos. This formula for the neutrino mass matrix is called type II seesaw formula [28].

3.2. Understanding large mixings for degenerate neutrinos

A major puzzle of quark lepton physics is the diverse nature of the mixing angles between quarks and leptons. Whereas in the quark sector the mixing angles are small, for the leptons they are large.

In order to understand the mixing angles, we have to study the mass matrices for the charged leptons and neutrinos. Since we can choose an arbitrary basis for either the charged leptons or the neutrinos without effecting weak interactions,

we will work in a basis where charged lepton mass matrix is diagonal. One can then look for the types of mass matrices for neutrinos that can lead to bi-large mixings and try to understand them in terms of new physics.

(i) *The case of normal hierarchy:* A neutrino mass matrix that leads to bi-large mixing in this case has the form:

$$M_\nu = m_0 \begin{pmatrix} \epsilon & \epsilon & \epsilon \\ \epsilon & 1 + \epsilon & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \quad (7)$$

where m_0 is $\sqrt{\Delta m_{ATM}^2}$. We have omitted order one coefficients in front of the ϵ 's. This leads to $\tan\theta_A \simeq 1$, $\Delta m_{\odot}^2 / \Delta m_A^2 \simeq \epsilon^2$ and also large solar angle. For the LMA I solution, we find the interesting result that $\epsilon \sim \lambda$ where λ is the Cabibbo angle ($\simeq 0.22$). This could be a signal of hidden quark lepton connection. In fact we will see below that in the context of a minimal SO(10) model, this connection is realized in a natural manner.

(ii) *The case of inverted hierarchy:* The elements of the neutrino mass matrix in this case have a slightly different pattern.

$$M_\nu = m_0 \begin{pmatrix} \epsilon & c & s \\ c & \epsilon & \epsilon \\ s & \epsilon & \epsilon \end{pmatrix}. \quad (8)$$

where $c = \cos\theta$ and $s = \sin\theta$ and it denotes the atmospheric neutrino mixing angle. An interesting point about this mass matrix is that in the limit of $\epsilon \rightarrow 0$, it has $L_e - L_\mu - L_\tau$ symmetry. One therefore might hope that if inverted hierarchy structure is confirmed, it may provide evidence for this leptonic symmetry and which can be an important clue to new physics beyond the standard model. However the fact that the solar mixing angle appears to be far from being maximal means that $L_e - L_\mu - L_\tau$ symmetry must be badly broken.

(iii) *Degenerate neutrinos:* In this case, there are two ways to proceed: one may add the unit matrix to either of the above mass matrices to understand large mixings or look for some dynamical ways by which large mixings can arise. It turns that if neutrinos are mass degenerate, one can generate large mixings out of small mixings[30,31] purely as a consequence of radiative corrections.

We will call this possibility radiative magnification.

Let us illustrate the basic mechanism for the case of two generations. The mass matrix in the $\nu_\mu - \nu_\tau$ sector[31] can be written in the flavor basis as:

$$M_F(M_R) = U(\theta) \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} U(\theta)^\dagger \quad (9)$$

where $U(\theta) = \begin{pmatrix} C_\theta & S_\theta \\ -S_\theta & C_\theta \end{pmatrix}$. This mass matrix is defined at the seesaw (GUT) scale, where we assume the mixing angles to be small. As we extrapolate this mass matrix down to the weak scale, radiative corrections modify it to the form[32]

$$\mathcal{M}_F(\mathcal{M}_Z) = \mathcal{R} \mathcal{M}_F(\mathcal{M}_R) \mathcal{R} \quad (10)$$

where $\mathcal{R} = \begin{pmatrix} 1 + \delta_\mu & 0 \\ 0 & 1 + \delta_\tau \end{pmatrix}$. Note that $\delta_\mu \ll \delta_\tau$. So if we ignore δ_μ , we find that the $\tau\tau$ entry of the $\mathcal{M}_F(\mathcal{M}_Z)$ is changed compared to its value at the seesaw scale. If the seesaw scale mass eigenvalues are sufficiently close to each other, then the two eigenvalues of the neutrino mass matrix at the M_Z scale can be same leading to maximal mixing (much like MSW matter resonance effect) regardless what the values of the mixing angles at the seesaw scale are. Thus at the seesaw scale can even be same as the quark mixing angles as a quark-lepton symmetric theory would require. We call this phenomenon radiative magnification of mixing angles. It requires no assumption other than the near degeneracy of neutrino mass eigenvalues and is a new way to understand large mixings.

This has recently been generalized to the case of three neutrinos[33], where assuming the neutrino mixing angles at the seesaw scale to be same as the quark mixing angles renormalization group extrapolation alone leads to large solar and atmospheric as well as small θ_{13} at the weak scale provided the common mass of the neutrinos $m_0 \geq 0.1$ eV. We find that while both the solar and atmospheric mixing angles become large, the θ_{13} parameter remains small (0.08).

An important prediction of this model is that the common mass of the neutrinos must be big-

ger than 0.1 eV, as already noted for the radiative magnification mechanism to work. This result can be tested in the proposed neutrinoless double beta decay experiments. It is within the range of values reported in ref.[5].

4. MINIMAL SO(10) GRAND UNIFICATION AND NEUTRINO MIXINGS:

The minimal grand unification model for neutrinos is the one based on the SO(10) group since all standard model fermions and the right-handed neutrino fit into the **16**-dimensional representation of SO(10), resulting not only in a complete unification of the quarks and leptons but also yielding possible relations between the quark and lepton mass matrices. One may therefore hope that the neutrino oscillation parameters might be predictable in an SO(10) theory.

There are two simple routes to realistic SO(10) model building. In the first class, one may have smaller representations for the Higgs fields like **10** and **16** multiplets[34]. In this case, one must necessarily introduce nonrenormalizable terms to the superpotential to implement the seesaw they break R-parity which then induces rapid proton decay at an unacceptable level.

An alternative is to introduce both **10** and **126** Higgs multiplets to give fermion masses. In this class of models, there is no need to invoke nonrenormalizable terms and also R-parity is an automatic symmetry of the model. This naturally prevents the baryon and lepton number violating terms that give rise to rapid proton decay and also guarantees a naturally stable supersymmetric dark matter. It is these class of minimal models[35,36,37] that we discuss here.

In SO(10) models of this type, the **126** multiplet contains two parity partner Higgs submultiplets (called $\Delta_{L,R}$) which couple to $\nu_L \nu_L$ and $N_R N_R$ respectively and after spontaneous symmetry breaking lead to the type II seesaw formula for neutrinos, which plays an important role in magnifying the neutrino mixings despite quark-lepton unification[36,37].

As we will see a further advantage of using **126** multiplet is that it unifies the charged fermion Yukawa couplings with the couplings that con-

tribute to righthanded as well as lefthanded neutrino masses, as long as we do not include non-renormalizable couplings in the superpotential. This can be seen as follows[35]: it is the set $\mathbf{10} + \overline{\mathbf{126}}$ out of which the MSSM Higgs doublets emerge; the later also contains the multiplets $(3, 1, 10) + (1, 3, \overline{10})$ which are responsible for not only lefthanded but also the right handed neutrino masses in the type II seesaw formula. Therefore all fermion masses in the model are arising from only two sets of 3×3 Yukawa matrices one denoting the $\mathbf{10}$ coupling and the other denoting $\overline{\mathbf{126}}$ couplings.

The $\text{SO}(10)$ invariant superpotential giving the Yukawa couplings of the $\mathbf{16}$ dimensional matter spinor ψ_i (where i, j denote generations) with the Higgs fields $H_{10} \equiv \mathbf{10}$ and $\Delta \equiv \overline{\mathbf{126}}$.

$$W_Y = h_{ij}\psi_i\psi_j H_{10} + f_{ij}\psi_i\psi_j \Delta \quad (11)$$

In terms of the GUT scale Yukawa couplings, one can write the fermion mass matrices (defined as $L_m = \bar{\psi}_L M \psi_R$) at the seesaw scale as:

$$M_u = \bar{h} + \bar{f}; \quad (12)$$

$$M_d = \bar{h}r_1 + \bar{f}r_2; \quad (13)$$

$$M_e = \bar{h}r_1 - 3r_2\bar{f}; \quad (14)$$

$$M_{\nu D} = \bar{h} - 3\bar{f}. \quad (15)$$

where \bar{h} , \bar{f} , $r_{1,2}$ are functions of GUT scale Yukawa couplings and mixing parameters discussed. These mass sumrules provide the first important ingredient in discussing the neutrino sector. To see this let us note that they lead to the following sumrule involving the charged lepton, up and down quark masses:

$$k\tilde{M}_l = r\tilde{M}_d + \tilde{M}_u \quad (16)$$

where k and r are functions of the symmetry breaking parameters of the model. It is clear from the above equation that smallquark mixings imply that the contribution the charged leptons to the neutrino mixing matrix i.e. U_ℓ in the formula $U_{PMNS} = U_\ell^\dagger U_\nu$ is close to identity and the entire contribution therefore comes from U_ν . Below we show that $U_n u$ has the desired form with θ_{12} and θ_{23} large and θ_{13} small.

4.1. Maximal neutrino mixings from type II seesaw

In order to see how the type II seesaw formula provides a simple way to understand large neutrino mixings in this model, note that in certain domains of the parameter space of the model, the second matrix in the type II seesaw formula can much smaller than the first term. This can happen for instance when V_{B-L} scale is much higher than 10^{16} GeV. When this happens, one can derive the sumrule

$$M_\nu = a(M_\ell - M_d) \quad (17)$$

This equation is key to our discussion of the neutrino masses and mixings.

Using Eq. (17) in second and third generation sector, one can understand how large mixing angle emerges.

Let us first consider the two generation case [36]. The known hierarchical structure of quark and lepton masses as well as the known small mixings for quarks suggest that the matrices $M_{\ell,d}$ for the second and third generation have

$$M_\ell \approx m_\tau \begin{pmatrix} \lambda^2 & \lambda^2 \\ \lambda^2 & 1 \end{pmatrix} \quad (18)$$

$$M_q \approx m_b \begin{pmatrix} \lambda^2 & \lambda^2 \\ \lambda^2 & 1 \end{pmatrix}$$

where $\lambda \sim 0.22$ (the Cabibbo angle). It is well known that in supersymmetric theories, when low energy quark and lepton masses are extrapolated to the GUT scale, one gets approximately that $m_b \simeq m_\tau$. One then sees from the above sumrule for neutrino masses Eq. (17) that there is a cancellation in the (33) entry of the neutrino mass matrix and all entries are of same order λ^2 leading very naturally to the atmospheric mixing angle to be large. Thus one has a natural understanding of the large atmospheric neutrino mixing angle. No extra symmetries are assumed for this purpose.

For this model to be a viable one for three generations, one must show that the same $b - \tau$ mass convergence at GUT scale also explains the large solar angle θ_{12} and a small θ_{13} . This has been demonstrated in a recent paper[37].

To see how this comes about, note that in the basis where the down quark mass matrix is diago-

nal, all the quark mixing effects are then in the up quark mass matrix i.e. $M_u = U_{CKM}^T M_u^d U_{CKM}$. Using the Wolfenstein parametrization for quark mixings, we can conclude that that we have

$$M_d \approx m_b \begin{pmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \quad (19)$$

and M_ℓ and M_d have roughly similar pattern due to the sum rule. In the above equation, the matrix elements are supposed to give only the approximate order of magnitude. As we extrapolate the quark masses to the GUT scale, due to the fact just noted i.e. $m_b - m_\tau \approx m_\tau \lambda^2$, the neutrino mass matrix $M_\nu = c(M_d - M_\ell)$ takes roughly the form:

$$M_\nu = c(M_d - M_\ell) \approx m_0 \begin{pmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix} \quad (20)$$

It is then easy to see from this mass matrix that both the θ_{12} (solar angle) and θ_{23} (the atmospheric angle) are large. It also turns out that the ratio of masses $m_2/m_3 \approx \lambda$ which explains the milder hierarchy among neutrinos compared to that among quarks. Furthermore, $\theta_{13} \sim \lambda$. A detailed numerical analysis for this model has been carried out in [37] and it substantiates the above analytical reasoning and makes detailed predictions for the mixing angles [37]. We find that the predictions for $\sin^2 2\theta_\odot \simeq 0.9 - 0.94$, $\sin^2 2\theta_A \leq 0.92$, $\theta_{13} \sim 0.16$ and $\Delta m_\odot^2 / \Delta m_A^2 \simeq 0.025 - 0.05$ are all in agreement with data. Furthermore the prediction for θ_{13} is in a range that can be tested partly in the MINOS experiment but more completely in the proposed long baseline experiments.

In conclusion, the progress in the field of neutrino mass has been phenomenal. A lot is now known about the masses and mixings but a lot of crucial informations are still missing (e.g. whether the neutrino is a Dirac or Majorana particle; sign of Δm_{23}^2 to name a few). These need to be probed so that finally one can say that we know as much about the leptons as we do about quarks. As far as their implications for new physics beyond the standard model are concerned, seesaw mechanism and the existence of the right handed neutrino are emerging as two

dominant ideas but as to the details of mixings, no clear solution has emerged yet. The introduction of the right handed neutrino not only makes the particle physics quark-lepton symmetric but it also makes the weak interactions asymptotically parity conserving. The possibility that neutrino mass owes its origin to grand unification is a tantalizing possibility. The next decade promises to be as exciting in neutrino physics as the past one.

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